Stochastic weather generator

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Joint work with: Jan Lennartsson

Outline

Motivation

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Weather generator

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- Weak sense similarity

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- Extremes

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Data

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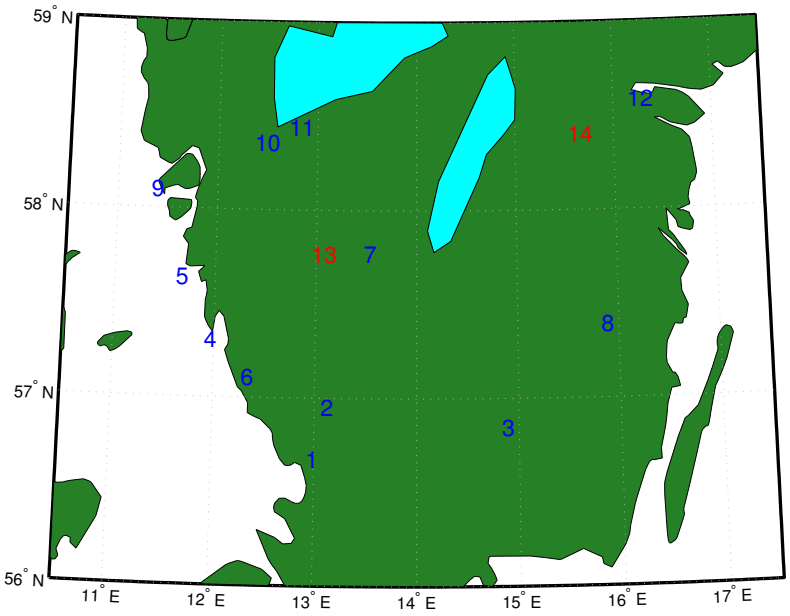
59° N
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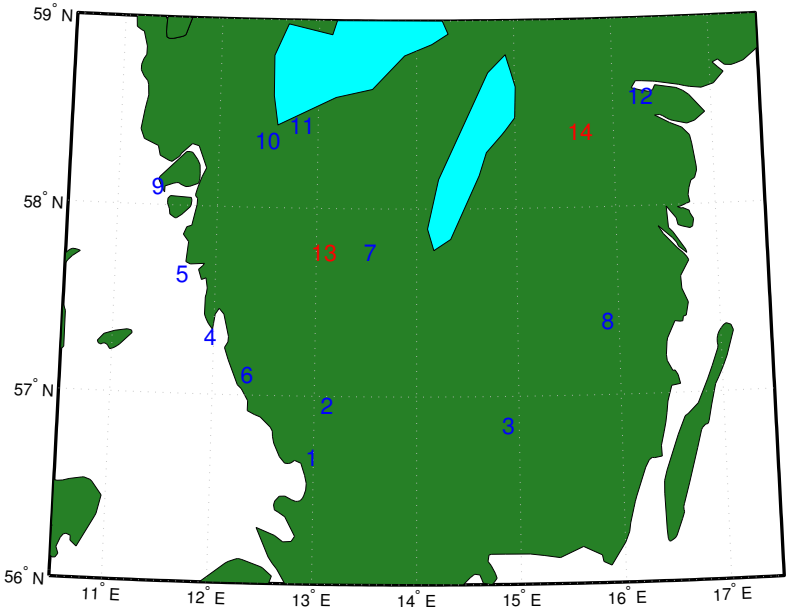

- location $\mathbf{s} = (x, y)$
- time t measured in days

Model specification

Latent Gaussian field $W(\mathbf{s}, t)$

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$$Y(\mathbf{s}, t) = F^{-1}_{\mathbf{s},t} \circ \Phi^+_\mu(W(\mathbf{s}, t))$$

where μ is mean of W - is amount of precipitation at location $\mathbf{x}$ and time t.

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Goal : Estimate mean and covariance function of W from observations of $W \vee 0$.

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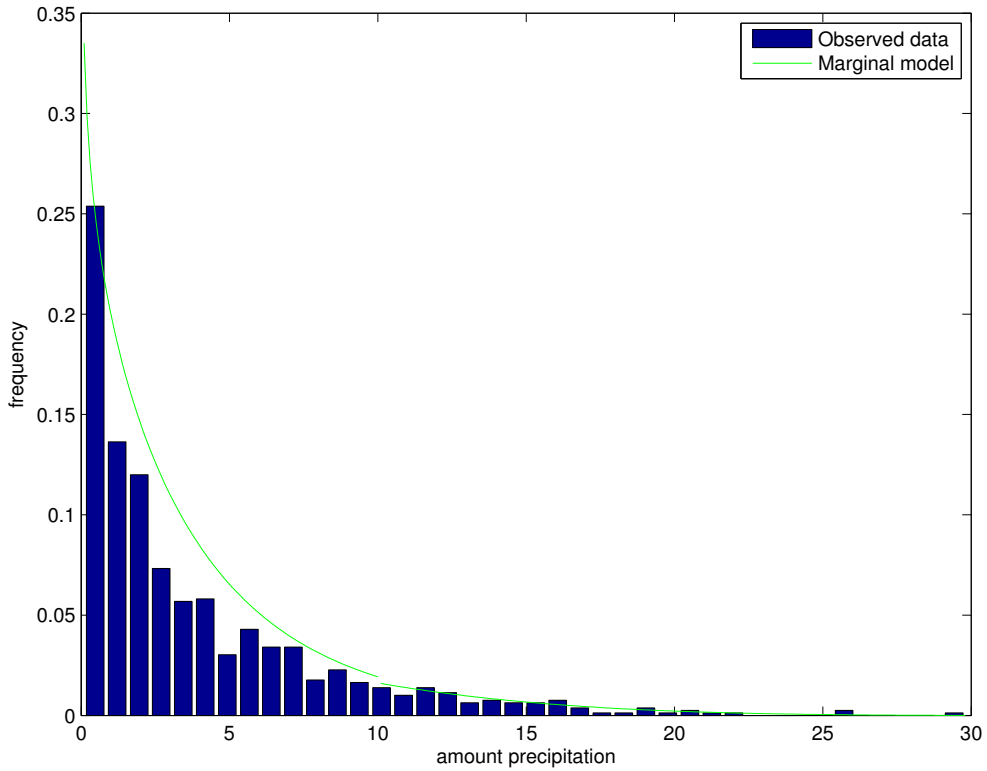

Fitting by modified likelihood

Mean function

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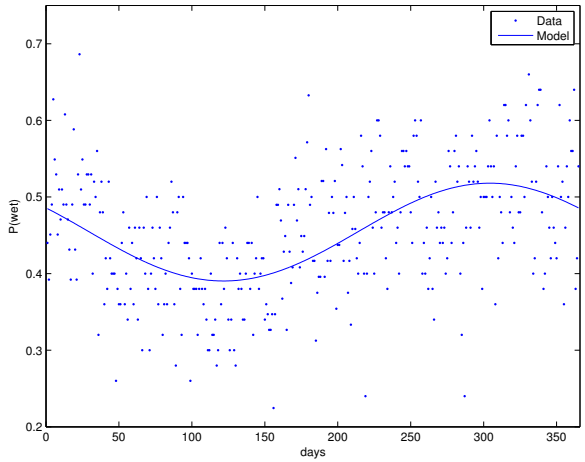

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Covariance function

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$$g(x; \boldsymbol{\mu}, \rho) = x\phi(x-\mu_1)\cdot\left[(\rho(x-\mu_2)+\mu_1)\cdot\Phi\left(\frac{\mu_1+\rho(x-\mu_2)}{\sqrt{1-\rho^2}}\right)+\right.$$

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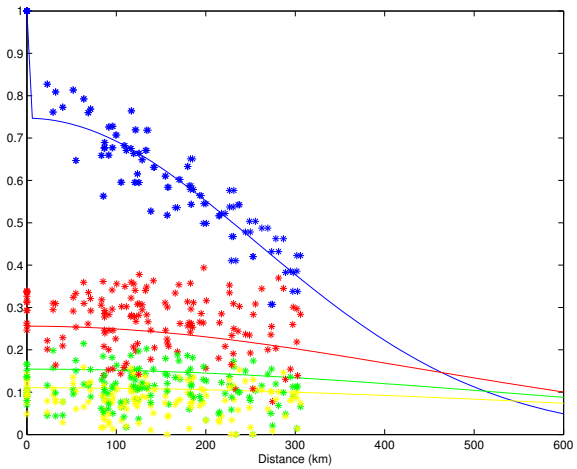

Validation

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How good is the stochastic generator?

- temporal
- spatial
- spatio-temporal

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Simulations!

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Temporal dependence structure

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- daily mean intensitites, conditional proportions, mean intensities
- monthly qq-plots
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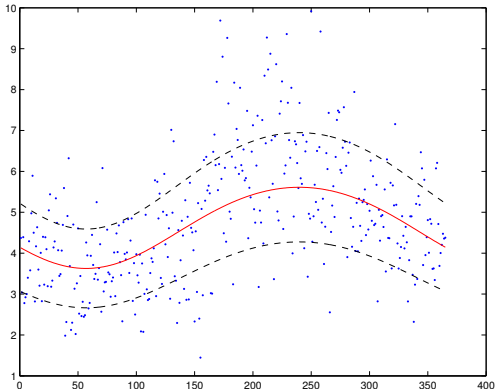

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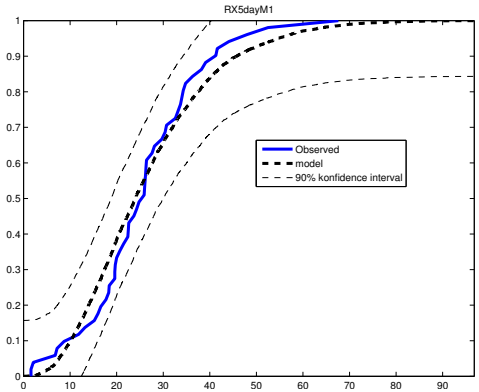

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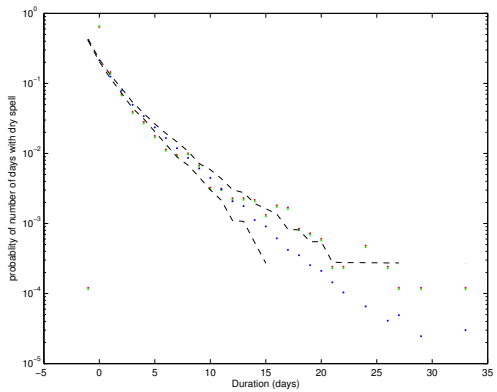

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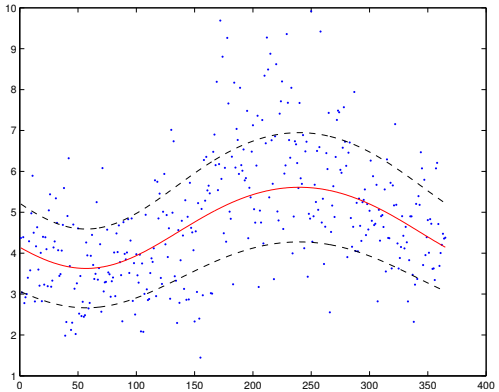

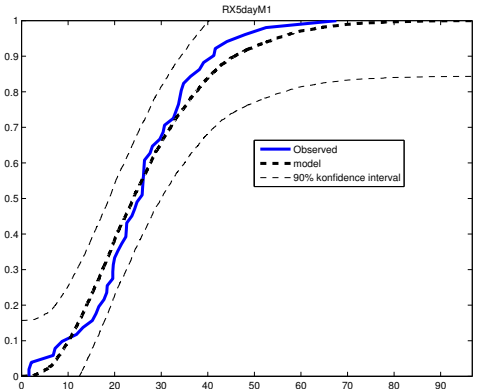

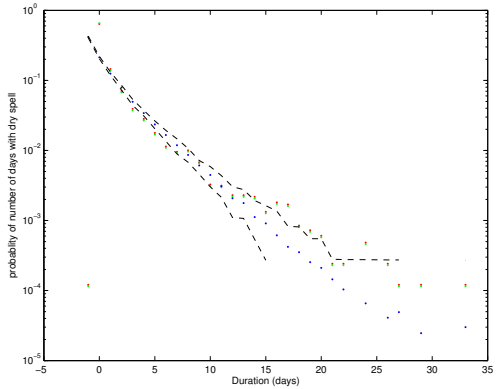

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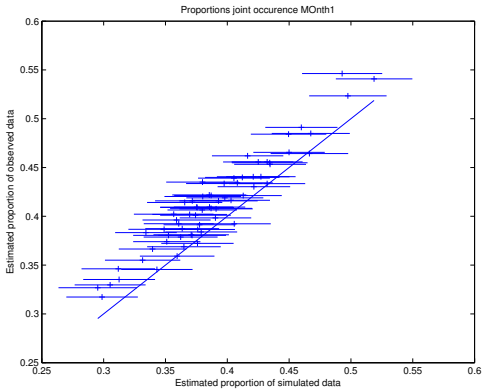

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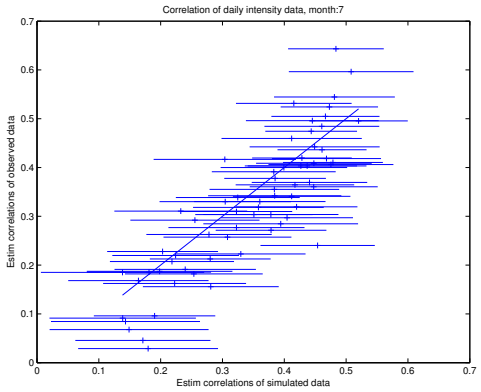

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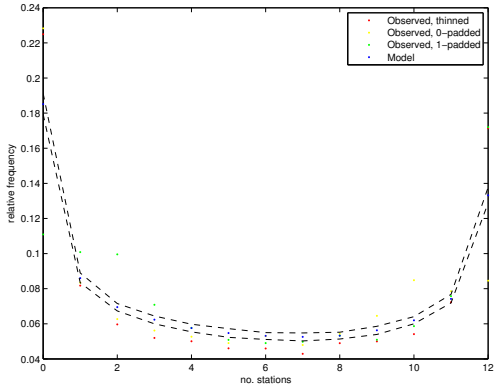

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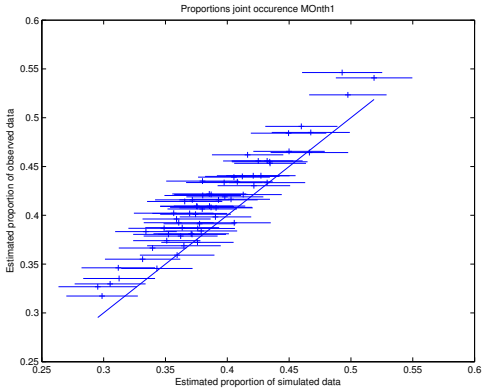

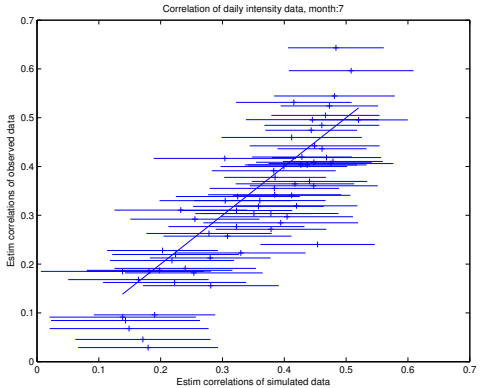

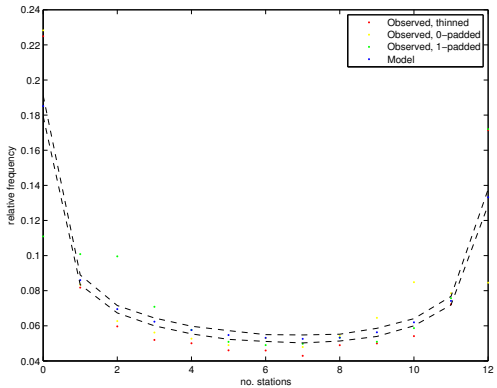

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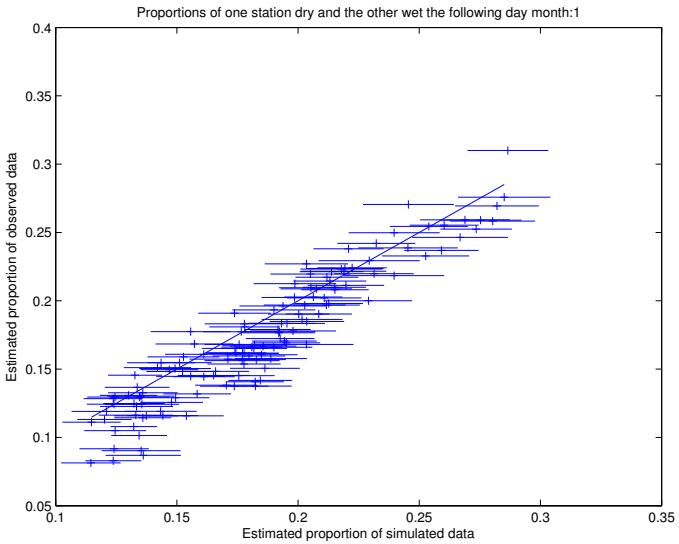

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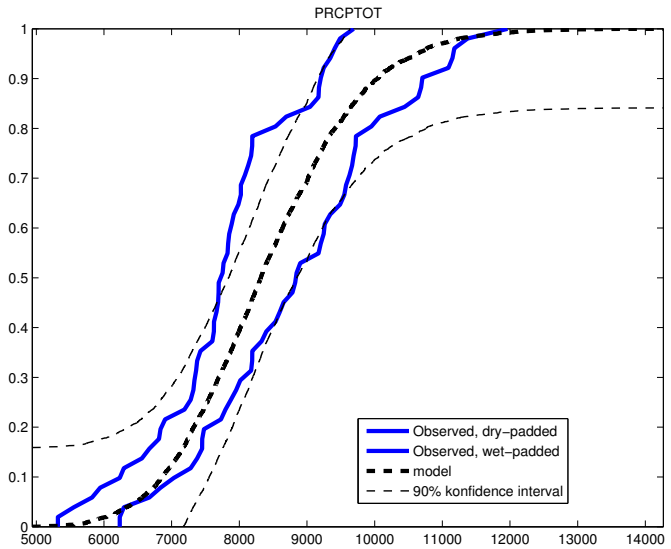

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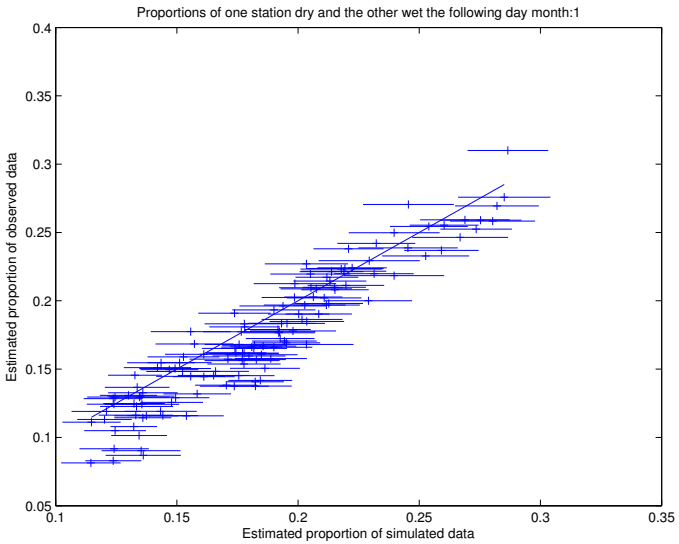

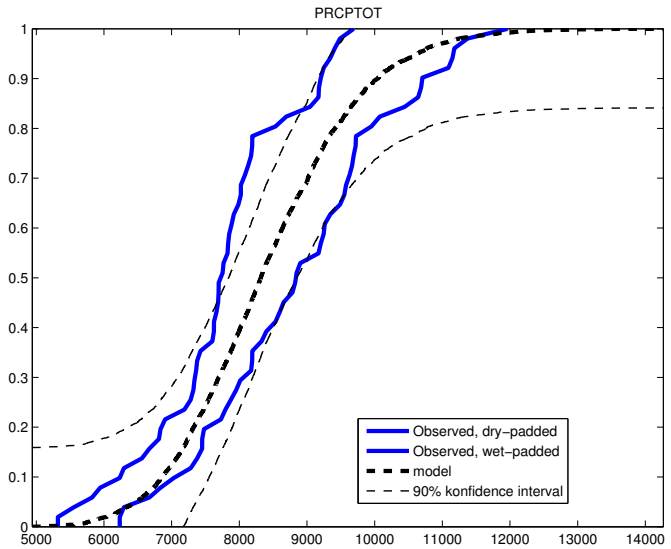

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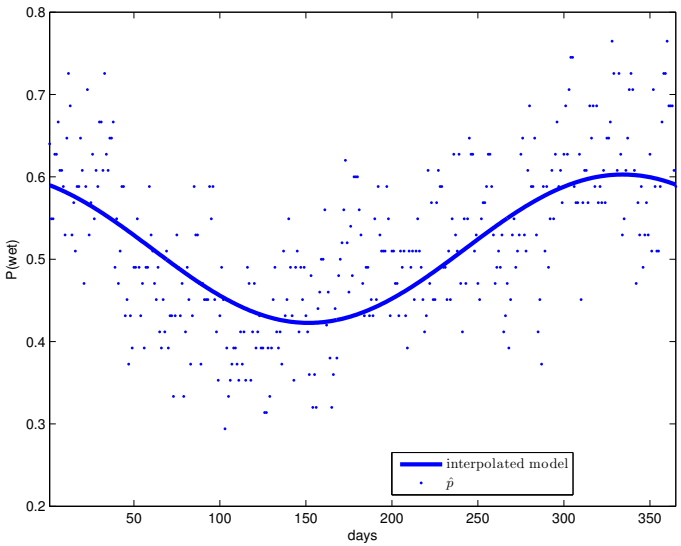

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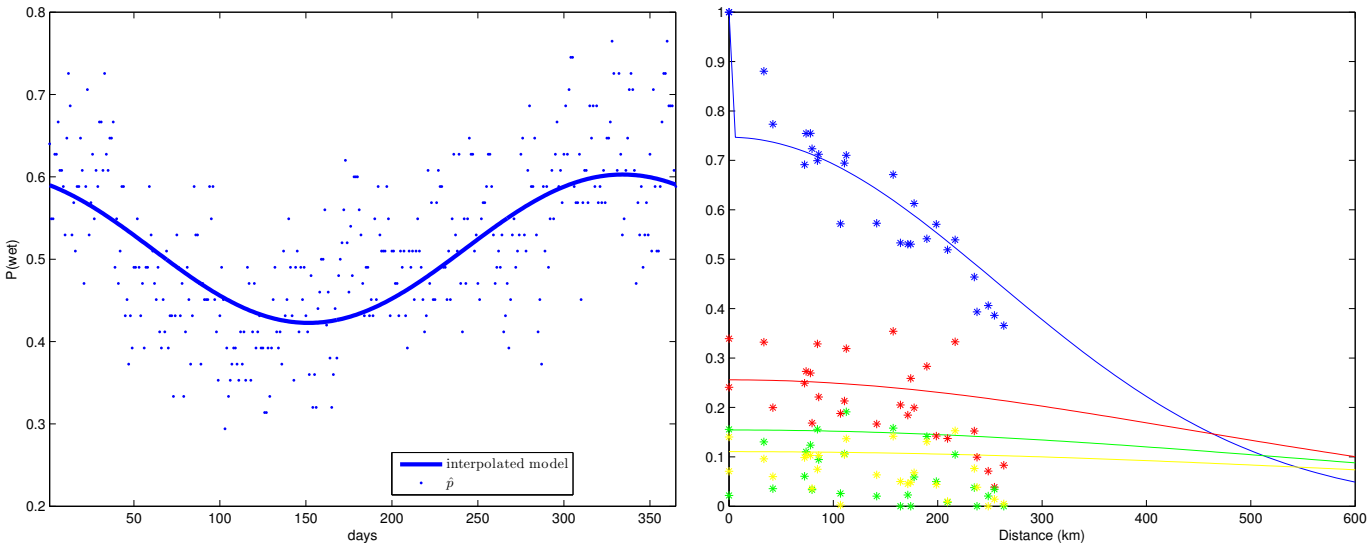

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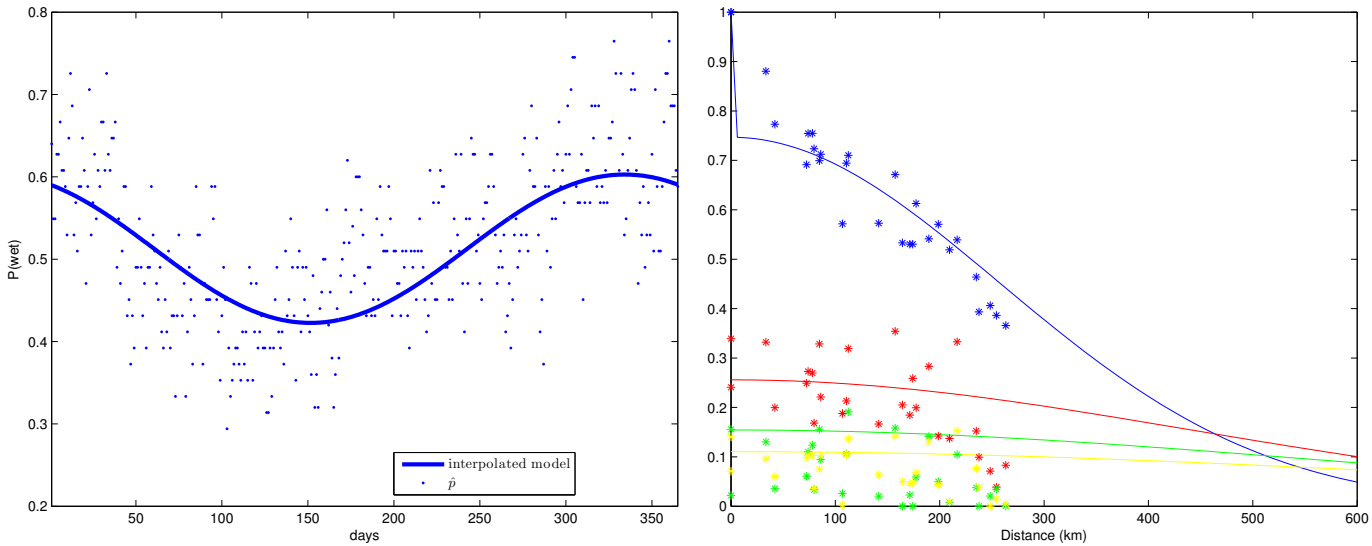

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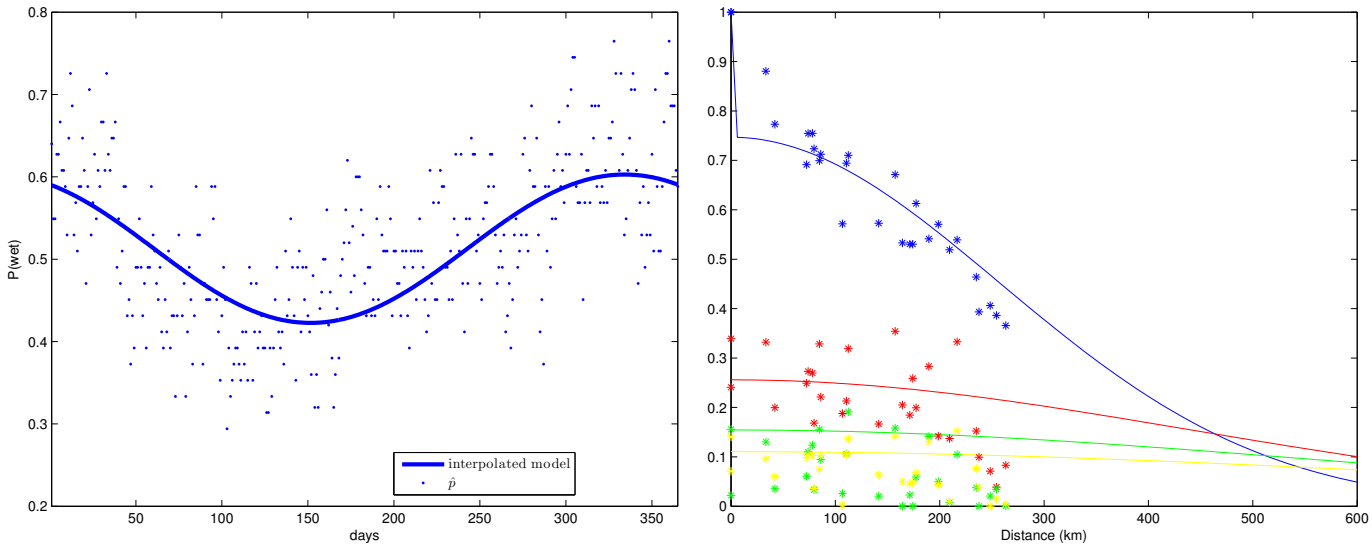

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