Conditional Modeling of Extreme Wind Gusts by Bivariate Brown-Resnick Processes

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Joint Work with Martin Schlather (University of Mannheim)
and Petra Friederichs (University of Bonn)

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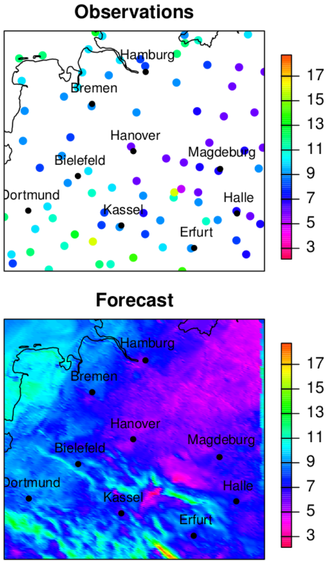
Extreme Wind Gusts

- wind gusts are strongly varying in space
- high uncertainty in forecasts, particularly for extreme wind gusts



Extreme Wind Gusts

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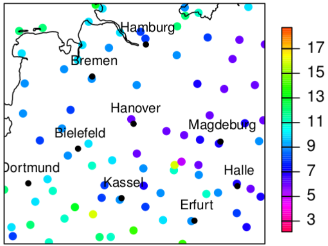
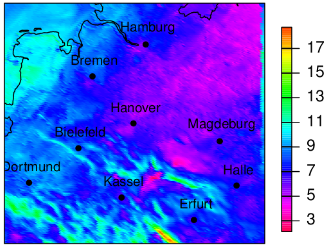
Goal:

Model for the (observed) wind gusts $V_{\max}^{\text{obs}}$ conditional on the forecast $V_{\max}^{\text{pred}}$

Observations



Forecast



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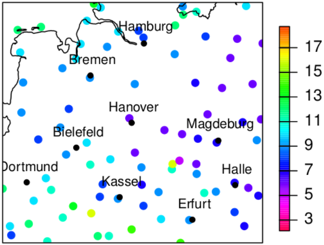

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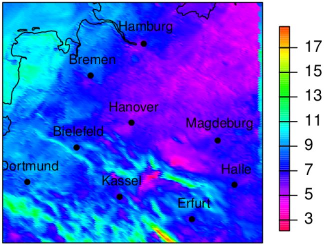

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Two Modelling Steps:

1. model for marginal distributions (at single location) of $V_{\max}^{\mathrm{obs}}$ & $V_{\max}^{\mathrm{pred}}$
2. dependence model
   $\rightsquigarrow$ bivariate stochastic process

Outline

Outline

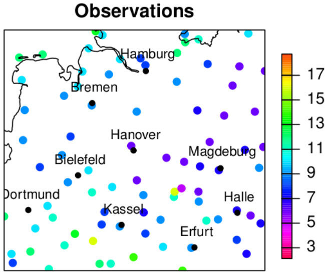
1. Marginal Model

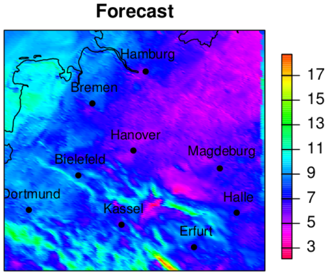
2. Dependence Model
   - Brown-Resnick Processes
   - Application to Data

The Data

Observation data:
for the maximal wind speed $\rightsquigarrow$ $V_{\text{max}}^{\text{obs}}$

- at 110 DWD stations in Northern Germany
- for 358 days (03/2011 – 02/2012)





The Data

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for the maximal wind speed $\rightsquigarrow V_{\text{max}}^{\text{obs}}$

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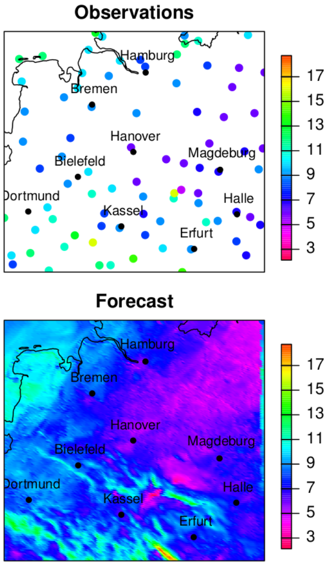
Forecast data:

from COSMO-DE EPS

- on a grid with mesh size 2.8 km covering Germany
- 20 ensemble members

1. for the maximal wind speed $\rightsquigarrow V_{\text{max}}^{\text{pred}}$
2. for the mean wind speed



Marginal Model for Extreme Wind Gusts

(single) **wind speed** $V(l, d)$ at location l and day d:

$$V(l, d) =_d \sqrt{\mathrm{Var}\{V(l, d)\}}\, V_0 + \mathbb{E}\{V(l, d)\}$$

with V_0 following some standardized distribution

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$\mathbb{E}\{V(l,d)\}$ and $\mathrm{Var}\{V(l,d)\}$:

- general weather conditions
- estimated from forecast

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maximal wind speed $V_{\max}(l, d)$ at location l and day d:

$$\mathbb{P}\left(\frac{V_{\max}(l,d) - \mathbb{E}\{V(l,d)\}}{\sqrt{\mathrm{Var}\{V(l,d)\}}} \leq x\right) \approx \exp\left(-\left(1 + \xi\frac{x-\mu}{\sigma}\right)_+^{-1/\xi}\right)$$

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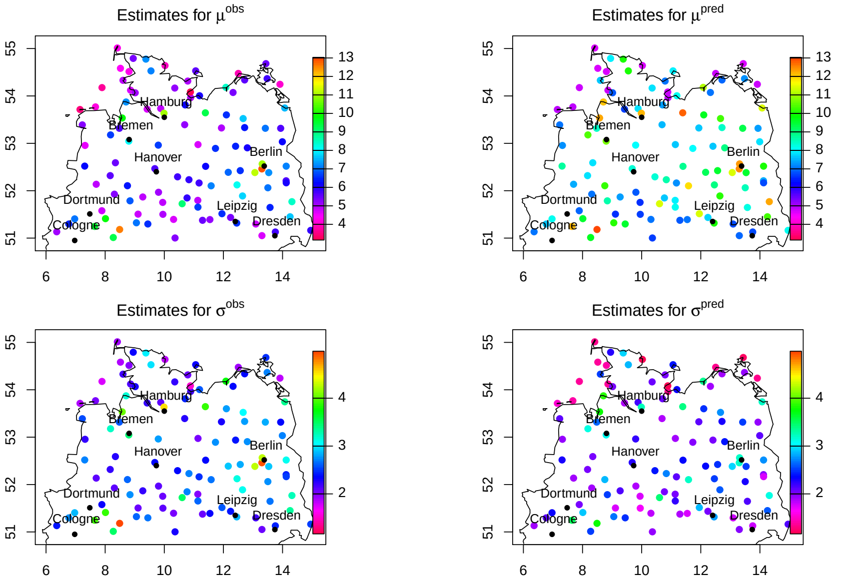
$\mathbb{E}\{V(l,d)\}$ and $\mathrm{Var}\{V(l,d)\}$:

- general weather conditions
- estimated from forecast

GEV parameters (ξ, μ, σ):

- ξ constant in space in time
- μ and σ vary spatially
- estimated via ML

GEV Parameters



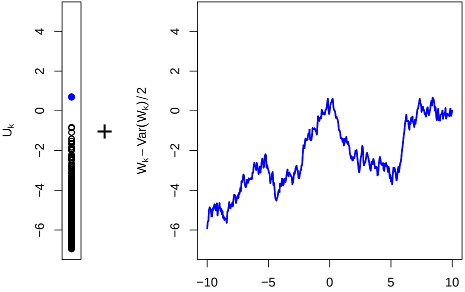
Outline

Spectral Representation

(Brown & Resnick 1977, Kabluchko, Schlather & de Haan 2009)

- $\{U_k\}_{k\in\mathbb{N}}$: PPP on $\mathbb{R}$ with intensity measure $e^{-u}\,\mathrm{d}u$ (magnitude)
- $W(\cdot)$: centered Gaussian process with stationary increments

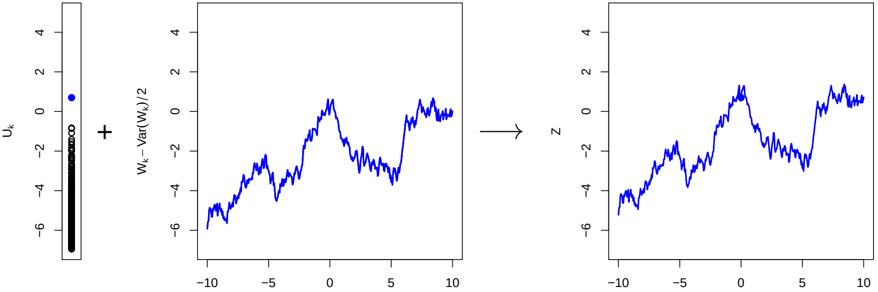
$$\left(W_k(\cdot) - \frac{\mathrm{Var}(W_k(\cdot))}{2}\right) \sim_{iid} \left(W(\cdot) - \frac{\mathrm{Var}(W(\cdot))}{2}\right) \quad \text{(spatial profile)}$$



Spectral Representation

(Brown & Resnick 1977, Kabluchko, Schlather & de Haan 2009)

$$Z(x) = \max_{k \in \mathbb{N}} \left(U_k + W_k(x) - \frac{\mathrm{Var}(W_k(x))}{2} \right), \quad x \in \mathbb{R}^d$$



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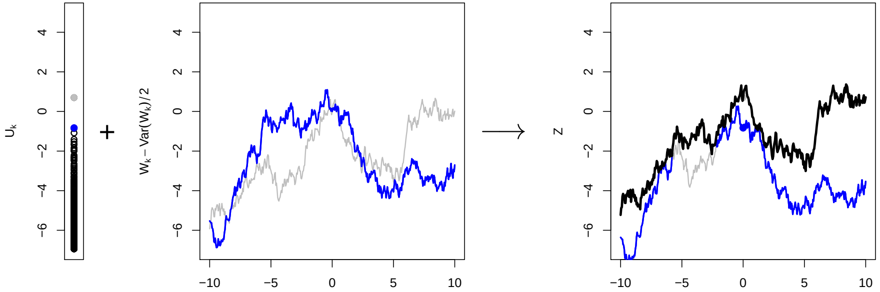

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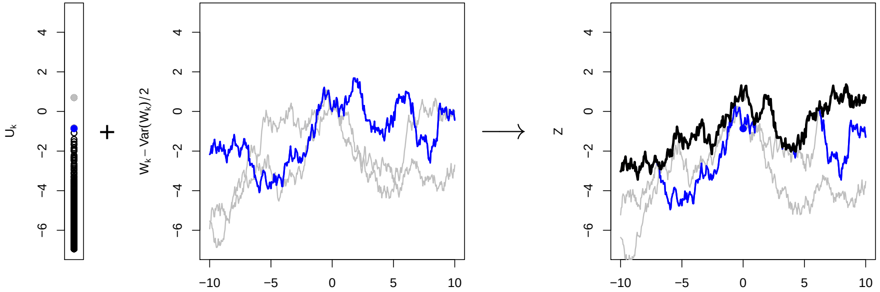

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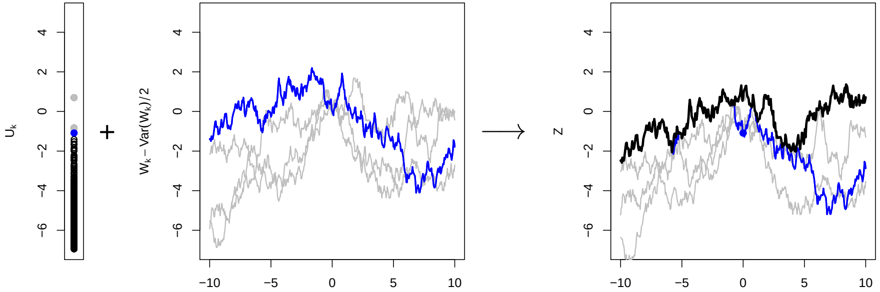

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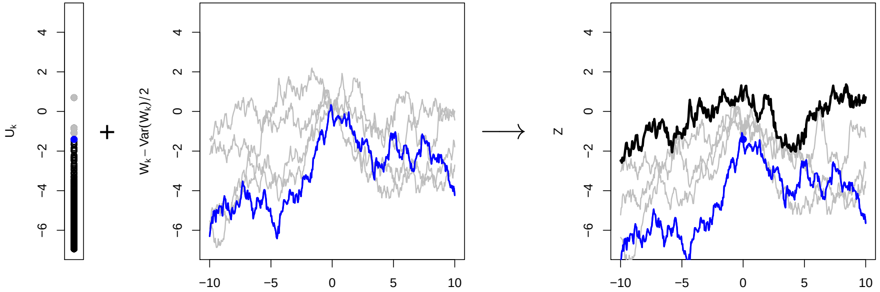

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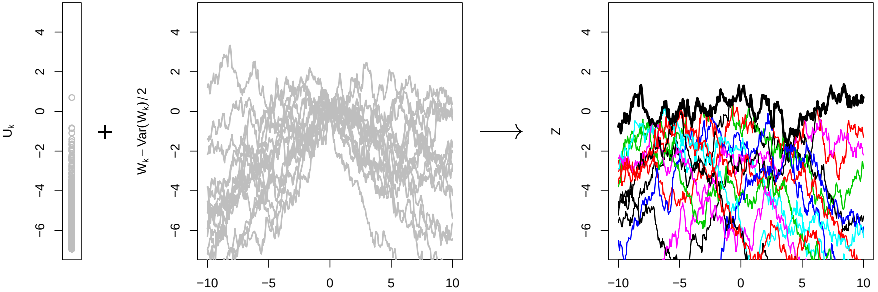

Z is max-stable and stationary!

Multivariate Generalization

(cf. Stucki & Molchanov, 2013, O. et al., 2013)

- $\{U_k\}_{k\in\mathbb{N}}$: Poisson point process with intensity $e^{-u}\,\mathrm{d}u$
- $W(\cdot)$: centered Gaussian process s.t.

variogram

$$\gamma(x_1, x_2) = (\mathrm{Var}(W \quad (x_1) - W \quad (x_2)))$$

depends on $x_1 - x_2 \in \mathbb{R}^d$ only

- $\{W_k(\cdot)\}_{k\in\mathbb{N}}$: independent copies of W

$$Z \quad (x) = \max_{k\in\mathbb{N}} \left(U_k + W_k \quad (x) - \frac{\mathrm{Var}(W_k \quad (x))}{2} \right), \quad x \in \mathbb{R}^d,$$

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pseudo-variogram

$$\gamma(x_1, x_2) = (\mathrm{Var}(W^{(i)}(x_1) - W^{(j)}(x_2)))_{i,j=1,2}$$

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Then,

- Z is max-stable and stationary (as bivariate process)
- law of Z depends on γ only

How to construct a valid pseudo-variogram?

Necessary condition:
(essentially) same behaviour of all components $\gamma_{ij}(h)$ as $\|h\| \to \infty$

Construction principle:

- $Y(\cdot)$: univariate Gaussian process with stationary increments and variogram γ^*
- $V(\cdot) = (V^{(1)}(\cdot), V^{(2)}(\cdot))$: bivariate stationary Gaussian process with covariance function

$$C(h) = \begin{pmatrix} C_{11}(h) & C_{12}(h) \\ C_{21}(h) & C_{22}(h) \end{pmatrix}$$

$W(\cdot) = (Y(\cdot) + V^{(1)}(\cdot), Y(\cdot) + V^{(2)}(\cdot))$ has a pseudo-variogram

$$\gamma(h) = \left(\gamma^*(h) + C_{ii}(0) + C_{jj}(0) - 2C_{ij}(h)\right)_{i,j=1,2}.$$

Outline

Full model

- **Reminder:**
  marginals of $V_{\text{max}}^{\text{obs}}$ and $V_{\text{max}}^{\text{pred}}$ are modelled by GEVs (parameters estimated via ML)

$V_{\text{max}}^{\text{obs}}$ (GEV)

$V_{\text{max}}^{\text{pred}}$ (GEV)

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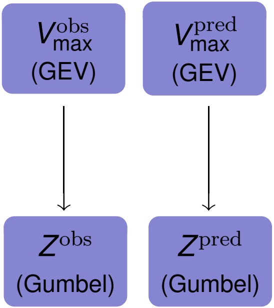

Full model

- **Reminder:**
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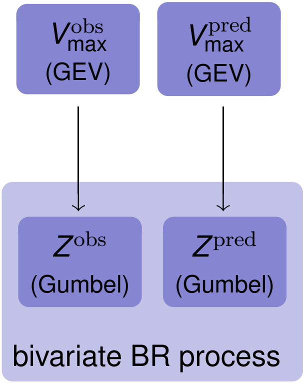
- data are transformed to standard Gumbel margins ($\rightsquigarrow Z^{\text{obs}}, Z^{\text{pred}}$)
- standardized observation and forecast are jointly modelled by bivariate BR process
  - dependence in space
  - dependence between observations and forecast

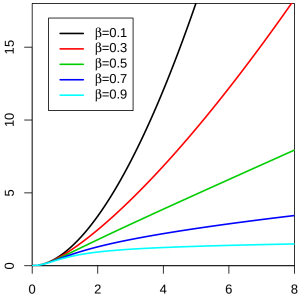


Bivariate Pseudo-Variogram Model

$$\gamma(h) = \left(\gamma^*(h) + C_{ii}(0) + C_{jj}(0) - 2C_{ij}(h)\right)_{i,j=1,2}.$$

- γ^*: variogram of power law type

$$\gamma^*(h) = \frac{\|h\|^2}{(1+\|h\|^2)^\beta}, \quad \beta \in (0,1)$$



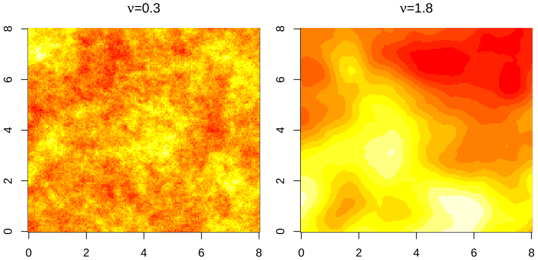
Bivariate Pseudo-Variogram Model

$$\gamma(h) = \left(\gamma^*(h) + C_{ii}(0) + C_{jj}(0) - 2C_{ij}(h)\right)_{i,j=1,2}.$$

- C: bivariate Matérn model (Gneiting et. al., 2010)

$$C_{ij}(h) = \rho_{ij}\sigma_i\sigma_j \frac{2^{1-\nu_{ij}}}{\Gamma(\nu_{ij})}(a\|h\|)^{\nu_{ij}} K_{\nu_{ij}}(a\|h\|)$$

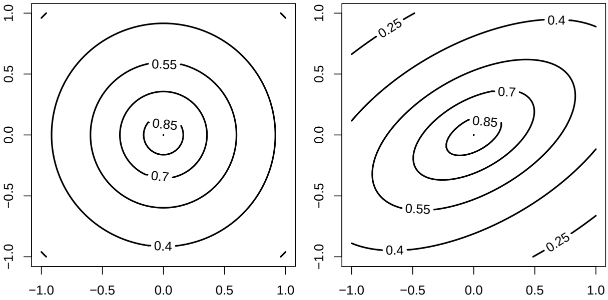
for $\sigma_1, \sigma_2 \geq 0$, $a, \nu_{11}, \nu_{22} > 0$, $\nu_{12} = (\nu_{11} + \nu_{22})/2$ and suitable ρ_{ij}



Bivariate Pseudo-Variogram Model

$$\gamma(h) = \left(\gamma^*(h) + C_{ii}(0) + C_{jj}(0) - 2C_{ij}(h)\right)_{i,j=1,2}.$$

- γ^*: $\gamma^*(h) = \|Ah\|^2 \Big/ (1 + \|Ah\|^2)^\beta, \quad \beta \in (0,1)$
- C: $C_{ij}(h) = \rho_{ij}\sigma_i\sigma_j 2^{1-\nu_{ij}}\Gamma(\nu_{ij})^{-1}\left(a\|Ah\|\right)^{\nu_{ij}}\mathcal{K}_{\nu_{ij}}\left(a\|Ah\|\right)$
- introduce anisotropy matrix A (dilation/rotation)

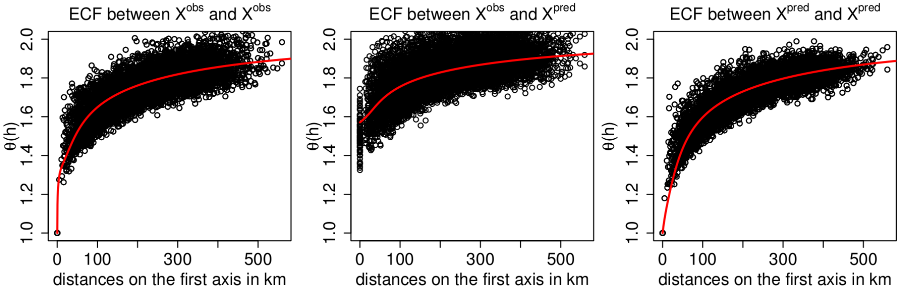


Extremal Coefficient Function

(cf. Schlather and Tawn 2003, Cooley, Naveau, Poncet 2009)

ECF between Z^{obs} and Z^{obs}

$$\mathbb{P}(Z^{\mathrm{obs}}(h) \leq z, Z^{\mathrm{obs}}(0) \leq z) = \mathbb{P}(Z^{\mathrm{obs}}(0) \leq z)^{\theta(h)}$$



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ECF between Z^{obs} and Z^{pred}

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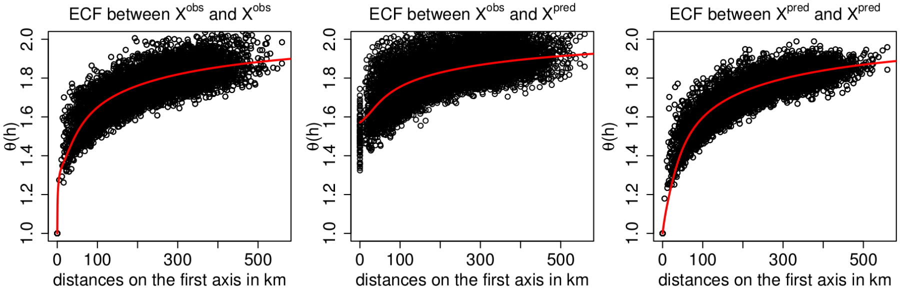

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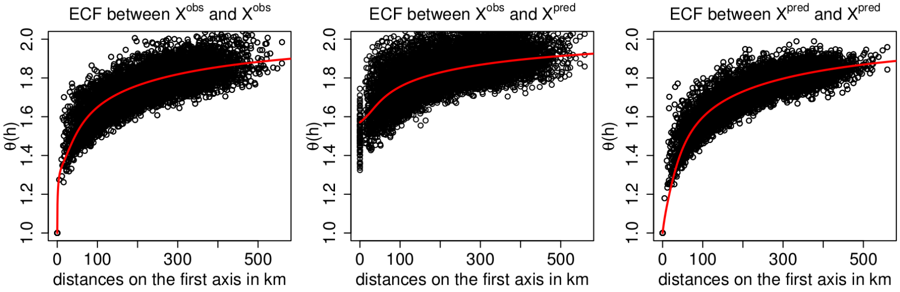

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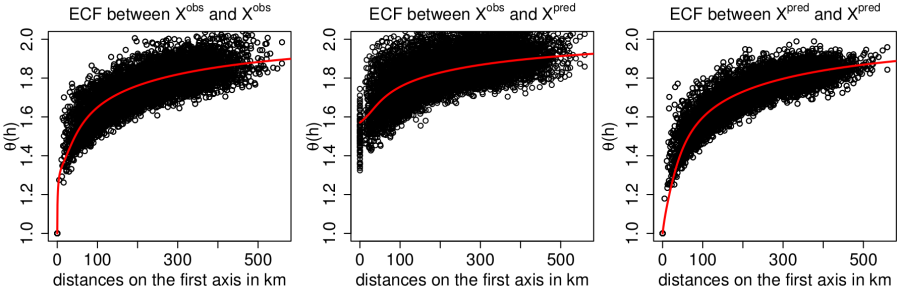
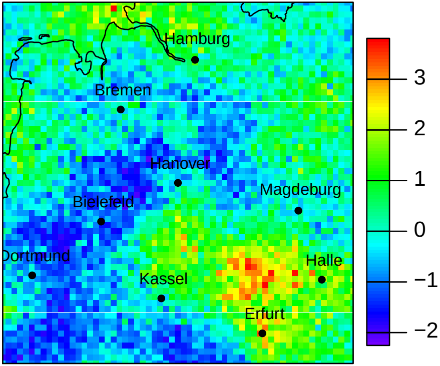


ECF for bivariate BR processes: $\theta_{ij}(h) = 2\Phi\left(\sqrt{\gamma_{ij}(h)}/2\right)$

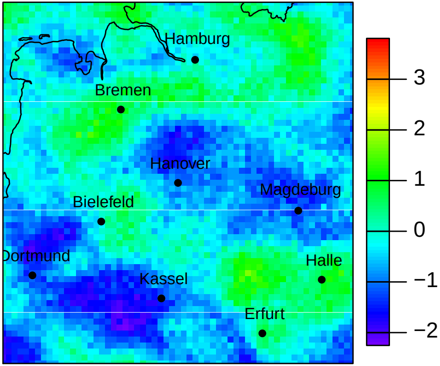
$\rightsquigarrow$ Fit ECF of bivariate BR process to estimated ECF!

Unconditional simulation of the Brown-Resnick process:

Realisation of X^{obs}



Realisation of X^{pred}



Post-Processing of the Forecast

Given Data

- historical observation & forecast data
- forecast for today

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Given Data

- historical observation & forecast data
  $\rightsquigarrow$ parameter estimation
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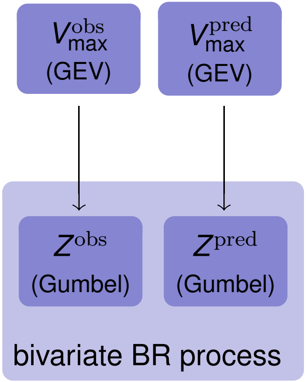
Post-Processing:

1. standardized forecast $V_{\max}^{\mathrm{pred}}$ to standard Gumbel margins $\rightsquigarrow Z^{\mathrm{pred}}$
2. simulate realizations of $Z^{\mathrm{obs}} \mid Z^{\mathrm{pred}}$
3. transform Z^{obs} from Gumbel to GEV margins $\rightsquigarrow V_{\max}^{\mathrm{obs}}$



Summary:

- bivariate Brown-Resnick processes model
  - dependence in space
  - dependence between forecast and observation
- flexible modelling via pseudo-variogram
- application to data shows promising results

Outlook/Open Questions:

- simulation of observations conditional on forecast
  $\rightsquigarrow$ conditional simulation of bivariate Brown-Resnick processes (Dombry, Eyi-Minko & Ribatet, 2013)
- downscaling (how to choose GEV parameters?)

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Verification

For every station, compare

“deviation” of COSMO-DE-EPS forecast from observation

and

“deviation” of post-processed forecast* from observation

*conditioned on forecast for the same station only

“Deviation” measure: Continuous Ranked Probability Score (CRPS)

Verification

For every station, compare

“deviation” of COSMO-DE-EPS forecast from observation

and

“deviation” of post-processed forecast* from observation

*conditioned on forecast for the same station only

“Deviation” measure: Continuous Ranked Probability Score (CRPS)

CRPS (cf. Gneiting & Raftery, 2007)

F: probability distribution (forecast) x: real value (observation)

$$\mathrm{CRPS}(F, x) = \int (F(y) - \mathbf{1}_{\{y \geq x\}})^2 \, \mathrm{d}y = \mathbb{E}|X - x| - \frac{1}{2}\mathbb{E}|X - X'|$$

for $X, X' \sim_{iid} F$ with $\mathbb{E}|X| < \infty$.

Verification

For every station, compare

“deviation” of COSMO-DE-EPS forecast from observation	and	“deviation” of post-processed forecast* from observation

*conditioned on forecast for the same station only

“Deviation” measure: Continuous Ranked Probability Score (CRPS)

Results:

For each of the 110 stations:

CRPS for post-processed forecast < CRPS for COSMO-DE-EPS,

on average by 13%.